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# Sparse Representation for Computer Vision and Pattern Recognition for Object detection

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***Abstract:*** Researchers in the domains of signal processing, image processing, computer vision, and pattern recognition have given sparse representation a lot of attention. In both theoretical research and real-world applications, sparse representation has a solid reputation. For sparse representation, numerous different algorithms have been put forth. There are many perspectives from which the taxonomy of sparse representation approaches can be investigated. For instance, the methods can be roughly divided into five groups based on the various norm minimizations used in sparsity constraints: sparse representation with lo-norm minimization, sparse representation with lp-norm minimization, sparse representation with l1-norm minimization, sparse representation with l2;1-norm minimization, and sparse representation with l2-norm minimization. Techniques from sparse signal representation are starting to have a considerable impact in computer vision, frequently on non-traditional applications where the objective is to extract semantic information as well as a compact, high-fidelity representation of the observed signal. In order to close this gap, the dictionary used is crucial. Unconventional dictionaries made from, or learned from, training samples themselves hold the key to achieving cutting-edge outcomes and giving semantic meaning to sparse signal representations. It also necessitates the development of new algorithmic and analytical tools to comprehend the superior performance of such unusual dictionaries.

**Keywords:** Sparse representation, Compressed sensing; computer vision; pattern, recognition; signal representations

## 1. Introduction

For the acquisition, representation, and compression of high-dimensional signals, sparse signal representation has proven to be a very effective tool. The major reason for this success is that significant classes of signals, such as audio and pictures, naturally have sparse representations with regard to fixed bases (such as Fourier or wavelet), or concatenations of such bases. Furthermore, it is possible to compute such representations with high fidelity using effective and efficient methods based on greedy pursuit or convex optimization [2]. While these achievements in traditional signal processing applications are encouraging, in computer vision, we are frequently more concerned with the content or semantics of an image than with a small, accurate representation. The question of whether sparse representation can be helpful for visual tasks at all is thus understandable. In painting [15], 16], background modelling [04], photometric stereo and image classification [17] have all used modifications and expansions of "1 minimization in recent years. The response has been mainly good. Sparsity as a prior produces cutting-edge results in practically all of these applications. Both algorithms and physical imaging systems have been influenced by sparsity and the creation of suitable dictionaries and projections [6]–[8], [21]. Although the images (or their characteristics) are naturally very high dimensional, in many applications, photos belonging to the same class exhibit degenerate structure, which contributes to the ability of sparse representations to expose semantic information. In other words, they are situated on, close to, or within low-dimensional subspaces, sub manifolds, or stratifications. We might anticipate that a typical sample has a highly sparse representation with regard to such a (potentially learnt) basis if a collection of representative samples is discovered for the distribution. 2 If computed correctly, such a sparse representation could inadvertently incorporate semantic information about the image. However, we often need to address the additional issue of how to properly select the foundation for representing the data in order to successfully employ sparse representation to computer vision tasks. In contrast to the typical signal processing environment, where a given basis with a desirable quality (such as being sufficiently incoherent) can be assumed.

In computer vision, we frequently have to work with a lexicon that is not necessary incoherent or learn one from sample images that are provided. We must therefore adapt the existing theory and sparse representation methods to new situations. A few illustrative examples of sparse representation in computer vision will be shown in this study. These examples show how vision issues could improve the theory of sparse representation, in addition to demonstrating how sparsity is a potent prior for visual inference. The ability to de-noise a dictionary by representing the test signal as a sparse linear combination of the training signals themselves can help solve some of the most difficult problems in computer vision, such as face recognition [02],[04],[13],[17], image super-resolution, motion and data segmentation, and motion analysis. We will first examine how this strategy produces straightforward and efficient face recognition algorithms. The face recognition example, in turn, reveals novel theoretical occurrences in sparse representation, which initially may seem unexpected.

Sparse representation has received a lot of attention recently, and there are numerous instances in a variety of domains where it is unquestionably advantageous and helpful [04, 12]. One such instance is picture classification, where thefundamental objective is to group the provided test image into a number of predetermined categories. It has been shown that, in terms of coefficients and training samples, natural pictures can be poorly represented. The sparse representation based classification (SRC) technique [20] makes the initial assumption that samples from the same subject can adequately represent the test sample. In more detail, SRC uses the linear combination of training samples to represent the test sample, computes the coefficients of the linear representation system for the sparse representation, and then determines the reconstruction residuals for each class using the sparse representation. Recent advances in sparse representation and learning have produced encouraging results in the field of visual recognition [19], which is concerned with the identification of objects and events in pictures and movies. Wright et al., for example, [20] proposed their well-known work on robust sparse representation facial recognition. The three main categories of visual recognition algorithms are unsupervised learning, supervised learning, and semi-supervised learning, depending on the availability of example vectors. Likewise, sparse representation and learning can be applied to supervised [20], semi-supervised [23], and unsupervised learning. In essence, sparse representation and learning can find hidden local neighboring structures with fewer data, whereas classic learning algorithms require more data to do the same objectives.

## 2. Graphical model interpretation of the sparse representation

### *2.1. Motivations*

Data clustering, subspace learning, and semi-supervised learning are just a few examples of the graph-based machine learning tasks that rely heavily on an informative graph, whether directed or undirected. Graphs depicting pairwise correlations between data samples are the basis for many well-liked spectral techniques to grouping [14]. ISOMAP [07], locally linear embedding (LLE) [12], and Laplacian Eigen maps (LEs) [11] are only a few examples of manifold learning methods that rely on graphs designed with varying objectives. In addition, the graph embedding framework [19] may be used to describe the most common subspace learning techniques, such as principal component analysis and linear discriminant analysis [10]. And many semi-supervised learning methods are propelled by regularizing graphs built over both labelled and unlabeled data [24]. Both the k-nearest-neighbor and the edge-joining methods are widely used in the aforementioned publications, and both are considered to be the de facto standard when it comes to graph creation "ball technique. The first one connects each data point with its k nearest neighbors, while the second one connects each data point with all samples within its surrounding. To maximize machine learning results, a graph should have the following properties.

1) Strong discerning ability.

The data from the same cluster or class are anticipated to be awarded substantial connection weights for data clustering and label propagation in semi supervised learning. However, the graphs created using those common methods frequently miss out on piecewise linear correlations between data samples belonging to the same class.

2) Sparseness. A sparse graph defining location relations can transmit the crucial information for classification, according to recent research on manifold learning [11]. Due to storage restrictions, a sparse graph is also the only viable option for large-scale applications.

3) An adaptive community.

It frequently occurs that the data are insufficient and not distributed uniformly, leading to diverse neighborhood structures for various data points. Both the k-nearest-neighbor and "-ball methods (generally) use a fixed global parameter to select the neighborhoods for all the data, hence they cannot be used in circumstances where an adaptive neighborhood is needed.

## 3. Categorization of sparse representation techniques

Techniques for sparse representation are categorized

Different vantage points can be used to categorize sparse representation theory. The existing sparse representation methods can be classified into various taxonomical groups using a variety of strategies because different methods each have their own unique motivations, ideas, and concerns. For instance, available sparse representation techniques can be divided into two main categories from the perspective of "atoms": naive sample-based sparse representation and dictionary learning-based sparse representation. However, sparse representation and learning methods can be broadly classified into three groups based on the availability of labels for "atoms": supervised learning, semi-supervised learning, and unsupervised learning methods. The sparse constraint causes the two communities of sparse representation methods—structure constraint based sparse representation and sparse constraint based sparse representation—to be separated. In addition, there are two main categories of representation-based classification methods in the area of image classification: holistic representation-based methods and local representation-based methods [21]. In more detail, while local representation-based methods only use training samples (or atoms) from each class or set of classes to represent the test sample, holistic representation-based methods use training samples from every class to represent the test sample. The majority of sparse representation techniques rely on holistic representation. The two-phase test sample sparse representation (TPTSR) method is a typical and representative local sparse representation technique [02]. The sparse representation approach can be divided into two categories when taking various methodologies into account: pure sparse representation and hybrid sparse representation, which enhances the pre-existing sparse representation approaches using other methodologies. According to the literature [04], convex relaxation, greedy algorithms, and combinational methods are roughly the three classes of sparse representation algorithms. The two categories of greedy algorithms and convex relaxation algorithms are typically used to categorize sparse decomposition algorithms in the literature [25] from the perspectives of sparse problem modelling and problem solving. The problems of sparse representation, however, can be divided into four optimization problems from the perspective of optimization: the smooth convex problem, the smooth non convex problem, the smooth convex problem, and the non-smooth nonconvex problem. Additionally, Schmidt et al. [22] reviewed a few optimization methods for resolving l1- norm regularization issues and roughly categorizes these techniques into three different optimization strategies: sub-gradient methods, unconstrained approximation methods, and constrained optimization methods. To fully comprehend the "taxonomy" of current sparse representation techniques described in this paper, refer to the supplementary file that is included with the paper. According to the analytical solution and optimization viewpoints, the available sparse representation methods in this paper are divided into four groups: constrained optimization strategy, proximity algorithm-based optimization strategy, and homotopy algorithm-based sparse representation. (1) Solving the sparse representation problem using the l0-norm minimization method is the main goal of the greedy strategy approximation. Due to the fact that this problem is NP-hard [17], the greedy strategy offers a rough solution to lessen the challenge. In order to arrive at the best overall solution, the greedy strategy iteratively seeks out the best local optimal solution. The greedy strategy approximation for the sparse representation method only selects the most k suitable samples, also known as k-sparsity, to approximate the measurement vector. (2) The main goal of the constrained optimization strategy is to find a suitable method for changing a non-differentiable optimization problem into a differentiable optimization problem by substituting a differentiable optimization term for the convex but un-smooth l1-norm minimization term. More specifically, on the initial unconstrained problem, the constrained optimization strategy replaces the l1-norm minimization term with an equal constraint condition. Given that l1-norm minimization is globally non-differentiable and that the original unconstraint problem can be reformulated into a differentiable problem with constraint conditions, the problem will become straightforward. (3) Proximal algorithms are effective tools for resolving non-smooth, constrained, large-scale, or distributed variations of the optimization problem [19]. The main goal of the proximity algorithm-based sparse representation optimization strategy is to reformulate the original problem into the specific model of the corresponding proximal operator, such as the soft thresholding operator, hard thresholding operator, and resolvent operator, and then to use the proximity algorithms to tackle the original sparse optimization problem. (4) The homotopy algorithm's general framework entails iteratively following the final desired solution from the starting point to the optimal point by gradually adjusting the homotopy parameter [24]. The homotopy algorithm is used to resolve the l1-norm minimization problem with the k-sparse property in homotopy algorithm-based sparse representation.

## 4. Classification Based on Sparse Representation

We first determine the sparse representation of a fresh test sample y from one of the classes in the training set using either. The test sample y can be easily attributed to a particular object class I if all of the nonzero entries in the estimate x1 are connected to columns of A from that class. Nevertheless, noise and modelling error could result in small nonzero entries linked to a variety of object classes. To solve this, a wide range of classifiers can be designed based on the global sparse representation. As an illustration, we can just assign y to the object class that has the largest entry in x1. Such heuristics, however, do not take advantage of the subspace structure connected to image data in face recognition. Instead, we categorize y based on how accurately all training samples for each object's coefficients reproduce y in order to better take advantage of this linear structure[02][04]

Let's say there are I classes. The characteristic function chosen by the coefficients linked to the ith class is IR- IRn. The entries in x that are connected to class I are the only nonzero entries in the new vector IRn for x. One can approximate the provided test sample y by writing it as using only the coefficients associated with the ith class. The object class that minimizes the residual between y and yi is then used to classify y based on these approximations:

$$\min_i \; r_i(\boldsymbol{y}) \doteq \|\boldsymbol{y} - A\,\delta_i(\hat{\boldsymbol{x}}_1)\|_2 \qquad (1)$$

### 5. Validation Based on Sparse Representation

Classifying a test sample requires first determining whether or not it is truly representative of one of the classes in the dataset. Real-world success for recognition systems requires their detection and rejection of invalid test samples, also known as "outliers." For instance, a face recognition system may be shown an image that is either not a face or of a person who is not in the database. The residuals are used not only for identification, but also for validation, in systems based on traditional classifiers like NN or NS. If the smallest residual is too large, the algorithm will reject the test sample. Each residual, however, is calculated independently of the other residuals and does not take into account any of the other object classes in the training data set; rather, it measures only the degree to which the test sample is like each individual class.

Coefficients x1 are calculated for all classes of images at once in the sparse representation paradigm. Essentially, it can use the combined distribution of all classes as proof. We argue that validation statistics based on coefficients x are preferable to those based on residuals. First, let's look at an illustration. [02]

### 6. Validation Based on Sparse Representation

We've shown how vision and machine learning can benefit from a sparse representation in an over complete dictionary made out of the samples themselves in the preceding sections. When working with data that has a linear or piecewise linear structure, such as in tasks like face recognition and motion segmentation, this concept can be quite helpful. However, for purposes like painting or denoising, the identification of the offered training samples is less relevant; they are merely a means to an end. In addition, it is less evident that images in one class should follow a single linear model in applications like generic image classification. It is likely that by optimizing a task-specific objective unction, more relevant dictionaries can be learned for such applications. In addition to making online processing more efficient, such dictionaries are typically substantially less in size than the initial training set. In this article, we will take a look at the many methods for learning such dictionaries, as well as their many uses in computer vision and image processing. [04]

#### A. Motivations

Sparse modelling, as we have seen, requires building effective representations of data as a (typically linear) combination of a small number of typical patterns (atoms) discovered within the data. State-of-the-art results have been achieved in many signal and image processing tasks [02]thanks to advances in the theory and practice of learning such collections of atoms (usually called dictionaries or codebooks), such as [4], and of representing the actual data in terms of them, such as [21]. If you are interested in a recent overview of the topic, we suggest reading [12]. The dictionary itself is crucial, and it has been repeatedly demonstrated that learnt dictionaries greatly outperform commercially available dictionaries like wavelets. Techniques now used to produce such dictionaries typically entail optimizing them in terms of the task to be performed, such as representation [18], denoising [4], and classification. Mutual coherence, cumulative coherence, and the Gram matrix norm of the dictionary are intrinsic properties of the dictionary that are related to theoretical results addressing the stability and consistency of the sparse solutions (active set of selected atoms) and the efficiency of the coding algorithms. These and similar goals can be optimized locally to learn dictionaries [7]. In this article, we introduce the fundamentals of dictionary learning and give some examples of the effectiveness of various algorithms for doing so.

#### B. Sparse Modeling for Image Reconstruction

Let $X \in R^{mxN}$ be a set of N column data vectors $X_j \in R^{m}$ and $D \in R^{mxK}$ of K atoms dictionary represented as columns $D_k$ R $^m$ .Each data vector xj will have a corresponding vector of reconstruction coefficients $\alpha_j$ $\in$ j 2 $R^K$, which we will treat as columns of a matrix [04]

$$A = [\alpha_1, \ldots, \alpha_N] \in \mathbb{R}^{K \times N}. \quad (2)$$

The penalty parameter determines the relative importance of the quadratic fitting term and the regularization term for each column of A in the cost function to be minimized in (11), where both terms are determined by the cost function (this parameter has been studied in [02],[04][12]). Convexity in A is achieved through the use of the '1-norm as an approximation to '0, which promotes sparse solutions [18]. While we discovered that the '0 penalty generally yields superior results for reconstruction, the '1 penalty is recommended for the classification tasks that will be discussed in the following section since it leads to more stable active sets. Motivated by either further estimating the '0 by '1 [19] or by considering a mixture of Laplacians prior for the coefficients in A and leveraging MDL principles, one may also substitute a (nonconvex) Lorentzian penalty function for these costs. Coordinate-descent-type optimization methods [2, 4, 18] have been proposed because (11) is not simultaneously convex in fA;Dg. The state-of-the-art has been achieved by extending these methods to multi scale dictionaries and color images, as shown in [22]. For a color image denoised using this method (see Fig. 1), as well as several more examples, comparisons, and applications in image demosaicing, image in painting, and image denoising. Figure also depicts an example of a dictionary that has been learned, with the value K equal to 256. In the multi scale example, the patch sizes range from 77 to 2020, with a sparsity of around one tenth of the signal dimension m, and over complete dictionaries with K > m are employed for image denoising.

Fig.1 Image classification via sparse modeling

### C. Image classification via sparse modeling

Data clustering, subspace learning, and semi-supervised learning are just a few examples of the graph-based machine learning tasks that rely heavily on an informative graph, whether directed or undirected. Graphs depicting pairwise correlations between data samples are the basis for many well-liked spectral techniques to grouping [4]. ISOMAP [07], locally linear embedding (LLE), and Laplacian eigen maps (LEs) [11] are only a few examples of manifold learning methods that rely on graphs designed with varying objectives [19]. In addition, the graph embedding framework may be used to describe the most common subspace learning techniques, such as principal component analysis [19] and linear discriminant analysis [10]. And many semi-supervised learning methods are propelled by regularizing graphs built over both labelled and unlabeled data. Both the k-nearest-neighbor and the edge-joining methods are widely used in the aforementioned publications, and both are considered to be the de facto standard when it comes to graph creation "ball technique. The first one connects each data point with its k nearest neighbors, while the second one connects each data point with all samples within its surrounding. To maximize machine learning results, a graph should have the following properties.

Fig.2 Simultaneously learning the dictionary and sensing

### D. Learning to Sense

As we have seen, learning over complete dictionaries that facilitate a sparse representation of the data as a liner combination of a few atoms from such dictionary leads to state-of-the-art results in image and video restoration and classification. The emerging area of compressed sensing (CS) (see [3], [18], [21] and references therein) has shown that sparse signals can be recovered from far fewer samples than required by the classical Shannon–Nyquist theorem. The samples used in CS correspond to linear projections obtained by a sensing projection matrix. It has been shown that, for example, a non-adaptive random sampling matrix satisfies the fundamental theoretical requirements of CS, enjoying the additional benefit of universality. A projection sensing matrix that is optimally designed for a certain class of signals can further improve the reconstruction accuracy or further reduce the necessary number of samples. In [13], the authors extended the formulation in to design a framework for the joint design and optimization, from a set of training images, of the nonparametric dictionary and the sensing matrix

**7. Conclusion:**

This paper introduces several sparse representation methods from different perspectives, covering topics such as their motivations, mathematical representations, and primary algorithms. We draw the following conclusions from the experimental findings summarized in Section IX. To begin, more in-depth research is required to determine the optimal regularization parameter for sparse representation, a difficult task. Adjusting the parameters in sparse representation algorithms is time-consuming and can be costly, as we can see by looking at how the value of the regularization parameter can significantly affect the performance of the algorithms. As this is a major problem, it is preferable to use sparse representation methods that are based on adaptive parameter selection, but few such methods have been proposed. Second, although sparse representation algorithms have achieved distinctively promising performance on some real-world databases, much work remains to be done to improve the precision of sparse representation based classification and to fortify the robustness of sparse representation. The recognition algorithms l1 is, homotopy, and TPTSR perform the best overall. When solving computer vision problems, sparse representation can be helpful in a number of ways. To begin, sparsity offers a robust prior for inference with high-dimensional visual data that contains intricate low-dimensional structures. Computational tools exist, such as '1-minimization that can be used to extract such structures, which can then be used to better exploit the data's semantics. Algorithms based on sparse representation can often achieve state-of-the-art performance, as we have seen in the few highlighted examples. Second, selecting the dictionary in such a way that sparse representations with respect to the dictionary accurately reveal the semantics of the data is crucial to actualizing this power. We can either do it implicitly by constructing the dictionary from data with linear or locally linear structure, or you can do it explicitly by optimizing various measures of the dictionary's in formativeness. Finally, the abundance of data and challenges in computer vision offer fresh examples for the theory of sparse representation, some of which call for additional mathematical analysis and justification. Our knowledge of sparse representation and computer vision for object detection and other application stands to benefit greatly from an examination of the resulting algorithms' efficiency.

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